Modified spontaneous symmetry breaking pattern by brane-bulk interaction terms

Francesco Coradeschi,[ab] **Stefania De Curtis**[b] **and Daniele Dominici**[ab]

[a]*Dipartimento di Fisica, Università di Firenze,*
I-50019 Sesto F., Firenze, Italy
[b]*I.N.F.N., Sezione di Firenze, I-50019 Sesto F., Firenze, Italy*
E-mail: coradeschi@fi.infn.it, decurtis@fi.infn.it, dominici@fi.infn.it

José R. Pelaez

Departamento de Física Teórica II, Universidad Complutense,
28040 Madrid, Spain
E-mail: jrpelaez@fis.ucm.es

ABSTRACT: We show how translational invariance can be broken by the vacuum that drives the spontaneous symmetry breaking of extra-dimensional extensions of the Standard Model, when delta-like interactions between brane and bulk scalar fields are present. We explicitly build some examples of vacuum configurations, which induce the spontaneous symmetry breaking, and have non trivial profile in the extra coordinate.

Contents

1. Introduction

The intense activity over the recent years in extra dimensional models has renewed the interest in the study of the vacuum configurations of such theories. Vacuum solutions with a non trivial behaviour in the extra coordinate have been investigated in simple five dimensional (5D) models where particles can be confined to a brane or live in the bulk, in particular to understand chirality properties or fermion masses and mixings, [1–3], or just to study the existence and stability of non trivial scalar configurations in simple $\lambda\phi^4$ theories on the circle or the orbifold [4–8], extending the pioneering papers on field localization in extra dimensions [9, 10]. In addition, brane terms are always generated by radiative corrections, even in the absence of tree level brane couplings [2]. The aim of this work is to show the inconsistency of assuming constant vacuum configurations to trigger a spontaneous symmetry breaking, without checking that they must also satisfy the constraints arising from interactions on the brane. If such constant configurations are not allowed, a non-trivial vacuum also has to break translational invariance. Hence one has to study the modification of the naive vacuum configuration in presence of such delta-like terms. Note that the effect of brane kinetic terms has been investigated for scalar, fermion and gauge theories in [2, 11–15] and they introduce subtleties of their own not related to our analysis, for this reason we will not use them to illustrate the effects we are interested in.

All that is needed for this vacuum translational invariance breaking to occur is that the condition for a constant non-vanishing vacuum coming from the brane has to be incompatible with the usual conditions coming from the bulk. There are many possibilities

for such localized terms: we could use brane masses, or self interactions for the bulk fields, or interactions between fields in the bulk and on the brane. Since our aim is just illustrative, we have chosen, for simplicity, a simple two-Higgs doublet model in five dimensions, assuming one Higgs in the bulk and the second one on the brane. Models of this type have been considered mainly from the phenomenological point of view as the simplest extensions of the Standard Model (SM) in five dimensions without supersymmetry [16, 17, 19, 20]. Nevertheless, the standard approach to 5D electroweak symmetry breaking with bulk Higgsses is through effective field theory. In this sense we could consider that this or similar models could also be obtained [18] as the low energy effective theory of a supersymmetric extra-dimensional extension of the SM [21–23]. Another reason for choosing two-Higgs models is that we have explicitly found that usually one assumes the existence of a constant vacuum solution for the bulk field, which does not depend on the extra coordinate, without discussing whether the two-Higgs potential admits such a solution. However, in a previous paper [24] we have already noticed that a constant solution in general does not exist, unless a particular relation among the quadrilinear couplings of the bulk and brane Higgs potential is satisfied. In this paper we provide analytic expressions for vacuum solutions and we build explicit examples with non trivial profiles corresponding to configurations which are absolute minima of the energy density. The examples are chosen to illustrate the extreme cases when the constant vacuum configuration is still a fairly good approximation, or when there is a sizable breaking of translational invariance. In addition they show that the procedure to obtain the solution in presence of other brane terms at a different fixed point is essentially similar.

In section 2 we review the $SU(2)_L \times U(1)_Y$ two-Higgs model in five dimensions, with the field Φ_1 propagating in the bulk and the field Φ_2 localized on the brane at $y = 0$. In section 3 we analyze the most general solutions, constant on the brane, of the equations of motion of the scalar fields in terms of Jacobi elliptic functions. In section 4 we explicitly build examples where these solutions have lower energy than the trivial $\Phi_1 = \Phi_2 = 0$ solution, showing that, indeed, non trivial vacuum configurations exist. These solutions lead to a spontaneous symmetry breaking with a pattern which is non standard, since the vacuum is not translationally invariant in the extra coordinate, and the vacuum expectation value of the bulk scalar field is not related to the Lagrangian parameters in the usual manner.

2. Delta-like interactions between brane and bulk fields

For illustration, let us consider a very simple 5D scalar model with an action containing both bulk and brane terms as follows:

$$S = \int_a^b dy \int d^4x \left\{ \mathcal{L}^{(5)} + \mathcal{L}^{(4)} \right\},$$
(2.1)

$$\mathcal{L}^{(5)} = \partial_M \Phi_1^\dagger \partial^M \Phi_1 - V^{(5)}(\Phi_1),$$
(2.2)

$$\mathcal{L}^{(4)} = \delta(y) \left[\partial_\mu \Phi_2^\dagger \partial^\mu \Phi_2 - V^{(4)}(\Phi_1, \Phi_2) \right],$$
(2.3)

where $M = \mu, 5$ and $a < 0 < b$. Note that Φ_1 has energy dimension $3/2$, whereas Φ_2 has dimension 1. There could be some other fields, but, for the following discussion, only Φ_1

and Φ_2 are relevant. In order to identify the vacuum state, we need to solve the equations of motion

$$(-\partial_y^2 + \Box)\Phi_1 = \frac{\delta V^{(5)}}{\delta \Phi_1} + \delta(y)\frac{\delta V^{(4)}}{\delta \Phi_1} \,, \tag{2.4}$$

$$\delta(y)\Box \Phi_2 = \delta(y)\frac{\delta V^{(4)}}{\delta \Phi_2} \,, \tag{2.5}$$

$$\sum_{\alpha=\Phi_1,\Phi_1^\dagger} \int d^4x \left[\left(\frac{\delta \mathcal{L}^{(5)}}{\delta \partial_y \Phi_\alpha} \delta \Phi_1 \right)_{y=b} - \left(\frac{\delta \mathcal{L}^{(5)}}{\delta \partial_y \Phi_\alpha} \delta \Phi_1 \right)_{y=a} \right] = 0\,. \tag{2.6}$$

The last term comes from the boundary conditions, and could also give rise to contributions that can be recast in terms of $\delta(y-a)$ and $\delta(y-b)$ functions and are thus similar to those that we will consider next. This said, and for simplicity, we will choose periodic boundary conditions so that eq. (2.6) is automatically satisfied. Nevertheless, the procedure to include another discontinuity is as simple as considering an additional boundary condition in the new fixed point. We will illustrate the procedure, which is very similar to the single brane case, in an example in section 4, where we impose a discontinuity in the bulk field derivative at $\pm\pi R$. Note that we consider, for simplicity, the circle with periodic boundary conditions, and that for the orbifold case, when the bulk field φ_1 is odd under the Z_2 orbifold projection, one could consider more general boundary conditions at $y = 0$ [25] than those above, which would deserve a separate and more general discussion.

The vacuum manifold corresponds to those solutions of the above equations of motion with minimum energy. Customarily, one considers constant solutions, so that the vacuum manifold corresponds to the minima of the potential, and, in particular, $\delta V^{(5)}/\delta \Phi_1 = 0$ and $\delta V^{(4)}/\delta \Phi_i = 0$ with $i = 1, 2$. However, we will show here that the presence of delta-like interactions between brane and bulk fields modifies the vacuum manifold in such a way that static field configurations are not allowed any more. We will show that this effect is non-perturbative and that even an infinitesimal value of such a coupling could avoid the presence of the naively expected pattern of spontaneous symmetry breaking on the brane.

In order to illustrate these effects we will concentrate on a model widely used in the literature, although our considerations are applicable to more general solutions of the kind described above (and probably involving other kind of fields like fermions, or more complicated interaction terms, as long as brane-bulk interactions are present).

2.1 An example within 5D extensions of the standard model

We consider a minimal 5D extension of the SM with two scalar fields. For the moment it is irrelevant whether the compactification is done on the $[-\pi R, \pi R]$ circle with periodic boundary conditions or in an orbifold S^1/Z_2, of length πR, since we are only interested in vacuum configurations. Of course, for oscillations around the vacuum the orbifold would lead to fields with definite y-parity.

In this simple model the $SU(2)_L$ and $U(1)_Y$ gauge fields and the Higgs field Φ_1 propagate in the bulk while the Higgs field Φ_2 lives on the brane at $y = 0$. The Lagrangian of

the gauge Higgs sector is given by (see [20] for a review)

$$\int_{-\pi R}^{\pi R} dy \int dx\, \mathcal{L}(x,y) = \int_{-\pi R}^{\pi R} dy \int dx \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} F_{MN}^a F^{aMN} + \mathcal{L}_{GF}(x,y) \right.$$
$$\left. + (D_M \Phi_1)^\dagger (D^M \Phi_1) + \delta(y)(D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V(\Phi_1, \Phi_2) \right\}, \quad (2.7)$$

B_{MN}, F_{MN}^a are the U(1)$_Y$ and SU(2)$_L$ field strengths and a is the SU(2)$_L$ index. The covariant derivative is defined as $D_M = \partial_M - ig_5 A_M^a \tau^a/2 - ig_5' B_M/2$. For simplicity we will consider a Higgs potential symmetric under the discrete symmetry $\Phi_2 \to -\Phi_2$, which is given by

$$V(\Phi_1, \Phi_2) = \mu_1^2 (\Phi_1^\dagger \Phi_1) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \delta(y) \left[\frac{1}{2} \mu_2^2 (\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \right. \quad (2.8)$$
$$\left. + \frac{1}{2} \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right],$$

where the dimensionalities of the couplings are: 1 for μ_1 and μ_2, -1 for $\lambda_1, \lambda_3, \lambda_4$ and λ_5, whereas λ_2 is dimensionless. The vacuum state manifold corresponds to configurations which are both energy minima and solutions of the following equations of motion:

$$(-\partial_y^2 + \Box)\Phi_1 = \mu_1^2 \Phi_1 + 2\lambda_1 (\Phi_1^\dagger \Phi_1)\Phi_1 \quad (2.9)$$
$$+ \delta(y) \left[\lambda_3 \Phi_1 (\Phi_2^\dagger \Phi_2) + \lambda_4 \Phi_2 (\Phi_2^\dagger \Phi_1) + 2\lambda_5 (\Phi_1^\dagger \Phi_2)\Phi_2 \right],$$
$$\delta(y)\Box\Phi_2 = \delta(y) \left[\mu_2^2 \Phi_2 + 2\lambda_2 (\Phi_2^\dagger \Phi_2)\Phi_2 + \lambda_3 (\Phi_1^\dagger \Phi_1)\Phi_2 \right.$$
$$\left. + \lambda_4 (\Phi_2^\dagger \Phi_1)\Phi_1 + 2\lambda_5 (\Phi_1^\dagger \Phi_2)\Phi_1 \right]. \quad (2.10)$$

However, one could naively think, and it is customarily assumed [16–20], that the extrema of the potential correspond to constant configurations $\Phi_1 = (0, v_1/\sqrt{4\pi R})$, $\Phi_2 = (0, v_2/\sqrt{2})$. Let us note, however, that if we substitute such constant solutions into the equations of motion above, we find

$$0 = v_1 \left(\mu_1^2 + 2\lambda_1 \frac{v_1^2}{4\pi R} \right), \quad (2.11)$$

$$0 = v_2 \left(\mu_2^2 + \lambda_2 v_2^2 + \frac{v_1^2}{4\pi R}(\lambda_3 + \lambda_4 + 2\lambda_5) \right), \quad (2.12)$$

$$0 = v_1 v_2^2 (\lambda_3 + \lambda_4 + 2\lambda_5). \quad (2.13)$$

If the trivial solutions $v_1 = v_2 = 0$ correspond to a minimum we have a trivial vacuum configuration and no spontaneous symmetry breaking. When implementing a spontaneous symmetry breaking one customarily builds the Lagrangian in such a way that $\mu_1^2 < 0$, $\mu_2^2 < 0$ and thus a minimum occurs for $v_1 \neq 0$ and $v_2 \neq 0$. Actually, this is the case of the two first conditions above. However, note that, for arbitrary values of λ_3, λ_4 and λ_5 such a solution is incompatible with the last condition, eq.(2.13), which is the one coming from the delta term in the Lagrangian. The usual constant solution is compatible with the brane

condition only if $\lambda_3 + \lambda_4 + 2\lambda_5 = 0$. This may come as a surprise since these constants parametrize the interaction of brane and bulk fields and are, in principle, independent. Thus, *even the tiniest value of an interaction with $\lambda_3 + \lambda_4 + 2\lambda_5 \neq 0$ destroys the usual ansatz of a translationally invariant vacuum state in the y direction.*

It is now easy to understand the remark we made in the introduction that all that is needed for this vacuum translational invariance breaking to occur is that the condition for a constant non-vanishing vacuum coming from the brane — eq.(2.13) in this case — has to be incompatible with the usual conditions coming from the bulk — eqs.(2.11) and (2.12). Actually, the position of the brane plays no role at all in eq. (2.13), which means that a similar braking effect could also be obtained with similar interaction terms located at other fixed points. In addition, any other brane term, like brane masses or self-interactions of the bulk fields, which are not included in our model for simplicity, as long as they would make the brane condition incompatible with the bulk conditions, would lead to the same kind of translational breaking.

For illustration, let us return to our simple model. In a previous work [24], for simplicity we required $\lambda_3 + \lambda_4 + 2\lambda_5 = 0$, which ensures that the minimum of the potential corresponds to the usual ansatz. In this way, the Higgs fields are expanded in the standard form

$$\Phi_1(x, y) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\omega^1 - i\omega^2) \\ \frac{1}{\sqrt{2}}(\frac{v_1}{\sqrt{2\pi R}} + h_1 - i\omega^3) \end{pmatrix}, \quad \Phi_2(x) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\pi^1 - i\pi^2) \\ \frac{1}{\sqrt{2}}(v_2 + h_2 - i\pi^3) \end{pmatrix}, \quad (2.14)$$

where $v_1 \equiv \sqrt{-2\pi R\mu_1^2/\lambda_1}$ and $v_2 \equiv \sqrt{-\mu_2^2/\lambda_2}$ are proportional to the vacuum expectation values of the scalar fields. Let us remark that we assume $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > -2\sqrt{2\pi R\lambda_1\lambda_2}$, otherwise the potential is not bounded from below.

In the literature this "constant ansatz" is sometimes assumed [20] without noting that the relation $\lambda_3 + \lambda_4 + 2\lambda_5 = 0$ is required, which thus limits the generality of the approach. We will see that, for certain choices of parameters, the assumption that the vacuum state is independent of y might still be a good approximation, although, as we will show, the vacuum expectation value of Φ_1 could be rather different from that of the constant case. Moreover, one could wonder what happens in the general case when $\lambda_3 + \lambda_4 + 2\lambda_5 \neq 0$ at tree level or if such a term was generated at higher orders from the different renormalization of the λ_3, λ_4 and λ_5 parameters. We will see that, by including such a term, the spatial invariance in the fifth dimension y is broken and non trivial vacuum configurations are obtained from solutions of eqs. (2.9) and (2.10).

3. Static solutions of the equations of motion

Following the previous discussion, in this section we will first search for solutions of the equations of motion that could play the role of the true vacuum. In section 4, we will study whether these solutions have a lower energy than the trivial vacuum so that they can trigger a spontaneous symmetry breaking. In particular we will look here for solutions that do not depend on the 4D space-time coordinates x, but still have a dependence on y.

For the sake of simplicity, and because we just want to illustrate the effects due to the presence of a $\delta(y)$ term, we will study the $\lambda_4 = \lambda_5 = 0, \lambda_3 \neq 0$ case. We can then recast the static vacuum solutions as

$$\langle \Phi_1(x,y) \rangle = \begin{pmatrix} 0 \\ \varphi_1(y) \end{pmatrix}, \quad \langle \Phi_2(x) \rangle = \begin{pmatrix} 0 \\ \varphi_2 \end{pmatrix}, \tag{3.1}$$

where $\varphi_1(y)$ is a real-valued field, and φ_2 a real constant. Therefore, the equations of motion, eqs. (2.9) and (2.10) for non-trivial vacuum solutions in this model, are reduced to

$$\partial_y^2 \varphi_1(y) - \varphi_1(y) \left[\mu_1^2 + 2\lambda_1 \varphi_1(y)^2 + \delta(y) \lambda_3 \varphi_2^2 \right] = 0, \tag{3.2}$$

$$\delta(y) \varphi_2 \left[\mu_2^2 + 2\lambda_2 \varphi_2^2 + \lambda_3 \varphi_1(y)^2 \right] = 0. \tag{3.3}$$

The above solutions have an associated energy density per unit volume:

$$\mathcal{H} = \int_{-\pi R}^{\pi R} dy \left[(\partial_y \varphi_1(y))^2 + \mu_1^2 \varphi_1(y)^2 + \lambda_1 \varphi_1(y)^4 + \delta(y) \left(\mu_2^2 \varphi_2^2 + \lambda_2 \varphi_2^4 + \lambda_3 \varphi_1(y)^2 \varphi_2^2 \right) \right]. \tag{3.4}$$

As usually done, we account for the presence of the δ-function by solving the δ-less equation

$$\partial_y^2 \varphi_1(y) - \varphi_1(y) \left[\mu_1^2 + 2\lambda_1 \varphi_1(y)^2 \right] = 0 \tag{3.5}$$

in the bulk regions $y < 0$ and $y > 0$ separately, and then connecting both pieces using the following boundary conditions

- continuity in $y = 0$:

$$\varphi_1(0^-) = \varphi_1(0^+) \equiv \varphi_1(0); \tag{3.6}$$

  As we commented in section 2, for the case of φ_1 odd under the Z_2 orbifold projection, more general boundary conditions at $y = 0$ [25] than the one above could be considered, that would require a discussion of their own.

- discontinuity of the first derivative in $y = 0$ with a gap $\lambda_3 \varphi_2^2 \varphi_1(0)$:

$$\varphi_1'(0^+) - \varphi_1'(0^-) = \lambda_3 \varphi_2^2 \varphi_1(0), \tag{3.7}$$

  where by eq. (3.3) we should have

$$\varphi_2^2 = -\frac{\mu_2^2}{2\lambda_2} - \frac{\varphi_1(0)^2 \lambda_3}{2\lambda_2}, \quad \text{with} \quad \varphi_2^2 > 0. \tag{3.8}$$

3.1 Solutions in the bulk

Let us solve eq.(3.5). Following [26, 5] we first multiply both sides by $\partial_y \varphi_1(y)$ and integrate in y, to get

$$\frac{1}{2} (\partial_y \varphi_1(y))^2 - \frac{1}{2} \mu_1^2 \varphi_1(y)^2 - \frac{\lambda_1}{2} \varphi_1(y)^4 = e_0, \tag{3.9}$$

where e_0 is a conserved quantity. Thus, integrating again

$$y - y_0 = \pm \int_{\varphi_1(y_0)}^{\varphi_1(y)} \frac{dt}{\sqrt{\mu_1^2 t^2 + \lambda_1 t^4 + 2e_0}}. \tag{3.10}$$

This integral can be solved analytically in terms of Jacobi elliptic functions [27, 28]. Such methods are well known, and thus we only provide the necessary steps to understand our notation. In particular, the exact solution depends on the nature of the roots of the polynomial

$$P_4(t) \equiv \lambda_1 t^4 + \mu_1^2 t^2 + 2e_0. \tag{3.11}$$

These are given by

$$t^2 = \frac{-\mu_1^2 \pm \sqrt{\mu_1^4 - 8e_0\lambda_1}}{2\lambda_1} \equiv \frac{-\mu_1^2}{2\lambda_1}\left(1 \mp \beta^2\right), \tag{3.12}$$

with

$$\beta^2 = \sqrt{1 - \alpha}, \quad \alpha = \frac{8e_0\lambda_1}{\mu_1^4}. \tag{3.13}$$

Hence, depending on the values of α, we have the following cases:

(A) $\alpha < 0$; $P_4(t)$ has two real and two complex solutions. We can therefore make use of the definition of the Jacobi elliptic $\mathrm{cn}(x, k^2)$ function:

$$\int_1^x \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 + k^2 t^2)}} = \mathrm{cn}^{-1}(x, k^2), \tag{3.14}$$

to rewrite eq. (3.10) as follows:

$$y - y_0 = \pm \frac{a}{\sqrt{N}} \int_{\frac{\varphi_1}{a}(y_0)}^{\frac{\varphi_1}{a}(y)} \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 + k^2 t^2)}}. \tag{3.15}$$

This is achieved by rescaling $t \to at$, so that

$$P_4(t) \to \lambda_1 a^4 t^4 + a^2 \mu_1^2 t^2 + 2e_0 \equiv N(1 - t^2)(1 - k^2 + k^2 t^2), \tag{3.16}$$

where

$$k^2 = \frac{1}{2}\left(1 + \frac{1}{\beta^2}\right), \ a^2 = \frac{-\mu_1^2}{2\lambda_1}(1 + \beta^2) > 0, \ N = \frac{-\mu_1^4}{2\lambda_1}\beta^2(1 + \beta^2) < 0. \tag{3.17}$$

In this way we finally get what we will call the "A type" solution

$$\varphi_1^A(y) = \pm \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1 + \beta^2} \ \mathrm{nc}\left(|\mu_1|\beta(y - y_0), \frac{1}{2}\left(1 - \frac{1}{\beta^2}\right)\right), \tag{3.18}$$

where we used the relation $\mathrm{cn}(ix, k^2) = \dfrac{1}{\mathrm{cn}(x, 1 - k^2)} \equiv \mathrm{nc}(x, 1 - k^2)$.

(B) $0 \leq \alpha \leq 1$, that is, $0 \leq \beta \leq 1$; in this case, $P_4(t)$ has four real solutions.

Again, we rescale $t \to at$; then we can match $P_4(t)$ to

$$P_4(at) \to N(1 - t^2)(1 - k^2 t^2), \tag{3.19}$$

which leads to a Jacobi elliptic $\mathrm{sn}(x, k^2)$ solution

$$\int_0^x \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 t^2)}} = \mathrm{sn}^{-1}(x), \tag{3.20}$$

with

$$k^2 = \frac{1 - \beta^2}{1 + \beta^2}, \qquad a = \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1 - \beta^2}, \qquad N = 2e_0 = \frac{\mu_1^4}{4\lambda_1}(1 - \beta^4) > 0, \qquad (3.21)$$

thus leading to what we will call "B1 type" solution

$$\varphi_1^{B1}(y) = \pm\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1 - \beta^2}\, \mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1 + \beta^2}(y - y_0), \frac{1 - \beta^2}{1 + \beta^2}\right), \qquad (3.22)$$

which is an oscillating function of y that satisfies $\varphi_1^{B1}(y_0) = 0$.

But we can also recast $P_4(t)$ as

$$P_4(at) \to N(1 - t^2)(t^2 - 1 + k^2), \qquad (3.23)$$

which now leads to a Jacobi elliptic $\mathrm{dn}(x, k^2)$ solution

$$\int_1^x \frac{dt}{\sqrt{(1 - t^2)(t^2 - 1 + k^2)}} = \mathrm{dn}^{-1}(x), \qquad (3.24)$$

by identifying,

$$k^2 = \frac{2\beta^2}{1 + \beta^2}, \qquad a = \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1 + \beta^2}, \qquad N = \frac{-\mu_1^4}{4\lambda_1}(1 + \beta^2)^2 < 0. \qquad (3.25)$$

This is what we will call a "B2 type" solution, which does not oscillate. It satisfies $\varphi_1^{B2}(y_0)/a = 1$, and can be written as

$$\varphi_1^{B2}(y) = \pm\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1 + \beta^2}\, \mathrm{dc}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1 + \beta^2}(y - y_0), \frac{1 - \beta^2}{1 + \beta^2}\right), \qquad (3.26)$$

where we have used the relation $\mathrm{dn}(ix, k^2) = \mathrm{dc}(x, 1 - k^2)$

(C) $\alpha > 1$. In this case β^2 is pure imaginary and $P_4(t)$ has no real solutions. We can rewrite eq. (3.10) as:

$$y - y_0 = \pm\int_{\tilde{\varphi}_1(y_0)}^{\tilde{\varphi}_1(y)} \frac{d\tilde{t}}{\sqrt{(\tilde{t}^2 - (1 + \sqrt{1 - \alpha}))(\tilde{t}^2 - (1 - \sqrt{1 - \alpha}))}}, \qquad (3.27)$$

where we have made the rescaling:

$$t \to \tilde{t} = \frac{\sqrt{2\lambda_1}}{|\mu_1|}t. \qquad (3.28)$$

This integral is not equal to the inverse of a Jacobi elliptic function, as those of the previous cases. However, although in a somewhat more tedious way, it can be solved by using the standard techniques for elliptic integrals [27, 28]. The general solution is:

$$\varphi_1^C(y) = \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{\frac{1}{2k^2 - 1}} \times \qquad (3.29)$$

$$\frac{\mathrm{dn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}(y - y_0), k^2\right) \pm \sqrt{1 - k^2}\, \mathrm{sc}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}(y - y_0), k^2\right)}{\mathrm{dn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}(y - y_0), k^2\right) \mp \sqrt{1 - k^2}\, \mathrm{sc}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}(y - y_0), k^2\right)},$$

with

$$k^2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{\alpha}}\right).$$ (3.30)

Let us now build the complete solutions of eq. (3.5) by imposing suitable boundary conditions in $y = 0$ and $y = \pm\pi R$.

3.2 Matching conditions

From integration, we initially have four free constants, two on the left side of the brane $y < 0$, that we call y_{0L} and β_L (y_{0L}, α_L in the case of type C solutions), and two more on the right side, $y > 0$, called y_{0R} and β_R (again, y_{0R}, α_R for solutions of type C). This fixes the shape of the function in the intervals, but, since the fields and their derivatives always appear squared in the action, there is an overall sign ambiguity, as it happens in the naive case with $\lambda_3 = \lambda_4 = \lambda_5 = 0$ where the vacuum in the bulk is given by either v_1 or $-v_1$.

Nevertheless, we are just looking for static minima of the action, which is symmetric under $y \leftrightarrow -y$. Hence the vacuum states must be even or odd under $y \leftrightarrow -y$, which implies $\beta_L = \beta_R \equiv \beta$. Also note that solutions which are antisymmetric under $y \leftrightarrow -y$ satisfy trivially the boundary condition (3.7); however if we require the continuity of $\varphi_1(y)$ in $y = \pi R$, its derivative has at least two nodes (one in the $(0, \pi R)$ region and the other in the $(-\pi R, 0)$ one), so it cannot correspond to a global minimum of the energy (as we have explicitly checked numerically). In conclusion we are only interested in *even* solutions and therefore $y_{0L} = -y_{0R} \equiv y_0$.

Then, apart from the overall sign arbitrariness, we are left with two constants β, y_0 that parametrize the space of possible candidates for vacuum configurations.

Furthermore our solutions should be of class C^1 except in $y = 0$, and possibly in $y = \pm\pi R$ where we could impose some additional boundary conditions. At $y = 0$ the left and right solutions should match each other according to eqs. (3.6), (3.7) and (3.8). The first one is automatically satisfied for even or odd functions, as in our case. If non trivial solutions do exist, then we must have $\mu_2^2 < 0$, so (3.8) tells us that, for $\lambda_3 > 0$, $\varphi_1(0)$ is bounded by $\varphi_1(0)^2 < -\mu_2^2/\lambda_3$. However, eq. (3.7) gives a relation between the two parameters β, y_0, that has to be solved numerically. All in all, that leaves us with just one free parameter. This one can be fixed if we impose an additional boundary condition on $y = \pm\pi R$. As we will see, the boundary condition could be as simple as requiring continuity of the first derivative in $\pm\pi R$, but other choices are possible. Similarly to the terms in eq.(2.6), one could even think of another delta-like interaction term localized in mirror branes in $y = \pm\pi R$.

In summary, by imposing the $y = 0$ boundary conditions in eqs. (3.6), (3.7) and (3.8), together with an additional boundary condition on $y = \pm\pi R$, one has sufficient constraints to fix, up to a global sign, the complete vacuum configuration in terms of the bulk solutions A, B1, B2, C detailed in the previous section. In general we found that a given choice of parameters does not allow the existence of all kind of solutions. Of course, the trivial solution $v_1 = v_2 = 0$ is always present, but it will not correspond to the true vacuum if one of the solutions described above has a lower energy. This will lead to a spontaneous

symmetry breaking with a pattern that does not correspond to the one customarily assumed in the literature, since the vacuum is not translationally invariant in the y variable. For some choice of parameters it can also happen that non trivial solutions cannot be found, so that there is no spontaneous breaking of symmetry.

In the next section we will show, with explicit examples, that for certain choices of the parameters, the non-trivial configurations do exist and have lower energy densities than the trivial one.

4. Examples of non-trivial vacuum configurations

We have seen how the solutions are basically fixed by the boundary conditions, once we know the Lagrangian parameters. Let us now remark that in the model we have considered in section 2 there are five independent parameters $\lambda_1, \lambda_2, \lambda_3$ and μ_1, μ_2. Note that, in the realistic case for the usual two-Higgs doublet one customarily chooses the parameters with the constraint $v_1^2 + v_2^2 = v^2 = (\sqrt{2} G_F)^{-1}$, which fixes the electroweak scale and one of the Lagrangian parameters in terms of the others. However, since now $\varphi_1(y)$ is not a constant, the integration over the fifth dimension does not factorize, and one can relate the electroweak scale to

$$v^2 \equiv 4 \int_0^{\pi R} \varphi_1(y)^2 \, dy + 2 \varphi_2^2 = 4 \int_0^{\pi R} \varphi_1(y)^2 \, dy - \frac{\mu_2^2 + \lambda_3 \, \varphi_1(0)^2}{\lambda_2} \,, \qquad (4.1)$$

where we have used eq. (3.8). Of course, in the $\lambda_3 \to 0$ limit, we recover the usual relation $v^2 = v_1^2 + v_2^2$.

In this work we are only interested in the vacuum, but, of course if the scalar φ_1 gets a non trivial vacuum profile, then the gauge vectors relative to the broken generators will also have non trivial profiles. As a consequence, the relation between the gauge boson mass and v will also be modified with respect to the standard case by a quantity of the order of the relative variation of the φ_1 along the extra dimension. For the precise relation one needs the explicit calculation of the profile of gauge modes, which lies beyond the scope of this work, but we plan to obtain in the future.

We will show that, depending on the boundary conditions on $y = \pm \pi R$, we can still find solutions for which the usual constant ansatz may be a good approximation, although the vacuum expectation value of the Φ_1 field on the $y = 0$ brane might be rather different from $v_1/\sqrt{4\pi R}$. In addition, there are solutions which change sizeably in the extra dimension and should not be approximated by a constant value. Both cases will be illustrated with the following examples.

4.1 Quasi-constant vacuum in the extra dimension

Let us impose, as a boundary condition, the continuity of the first derivative of $\varphi_1(y)$ in $y = \pm \pi R$. The periodicity, moreover, identifies the point πR with the point $-\pi R$; so, what we require is: $\varphi_1'(\pi R) = \varphi_1'(-\pi R)$. But since φ_1' is an odd function, then $\varphi_1'(-\pi R) = -\varphi_1'(\pi R)$ also comes true, so we conclude that $\varphi_1'(\pi R) = 0$, that is, πR is a maximum or
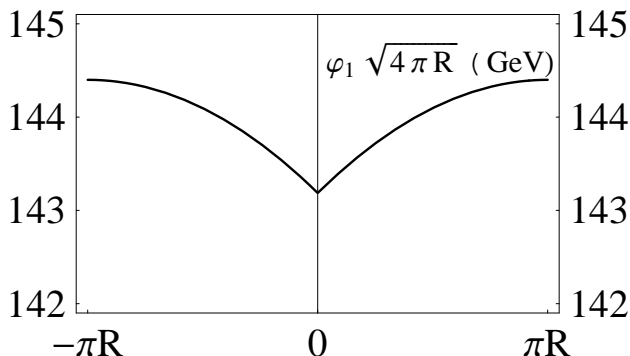
Figure 1: Vacuum configuration for the choice of parameters of section 4.1. Note that, by taking it as constant, (as it is for $\lambda_3 = 0$), might be a good approximation, since its variation from $y = 0$ to $y = \pi R$ is less than 1%. However, $\varphi_1(0)\sqrt{4\pi R} \simeq 233\,\mathrm{GeV}$ when $\lambda_3 = 0$, instead of $\varphi_1(0)\sqrt{4\pi R} \simeq 143\,\mathrm{GeV}$ here.

a minimum for $\varphi_1(y)$. Let us then make a simple choice of parameters:

$$\pi R = (1\mathrm{TeV})^{-1}, \quad \mu_1 = 165 \text{ GeV}, \quad \lambda_1 = 0.5 \times 2\pi R, \quad \lambda_2 = 1, \quad \lambda_3 = 0.85 \times 2\pi R. \quad (4.2)$$

Since we require $v = 246\,\mathrm{GeV}$ in eq.(4.1), apart from a global sign, there is only one continuous solution of eq. (3.7), that turns out to be of the B1-type, and can be written as follows:

$$\varphi_1^{B1}(y) = +\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1-\beta^2}\,\mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1+\beta^2}(y-y_0), \frac{1-\beta^2}{1+\beta^2}\right), \quad y > 0, \quad (4.3)$$

$$\varphi_1^{B1}(y) = -\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1-\beta^2}\,\mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1+\beta^2}(y+y_0), \frac{1-\beta^2}{1+\beta^2}\right), \quad y < 0, \quad (4.4)$$

with $\mu_2 \simeq 220\,\mathrm{GeV}$, $\beta \simeq 0.79$ and $y_0 \simeq 0.012\,\mathrm{GeV}^{-1}$. Here, for definiteness, we have taken the sign in front of the $y > 0$ solution to be positive, but of course, there is another solution with a global sign difference that will have the same energy.

The energy density can be calculated using eq. (3.4); we find that it is equal to $-(179\text{ GeV})^4$, which is less than the $(0\text{ GeV})^4$ associated with the trivial static solution, thus confirming the fact that we are in presence of a spontaneous symmetry breakdown. Actually, since there are no other solutions, the one we have found, shown in figure 1 corresponds to a global minimum and can be identified with the true vacuum. As it can be seen from the figure, a constant solution in this case would be an adequate approximation, since the difference between $\varphi_1(0)$ and $\varphi_1(\pi R)$ is less than 1%. However, we should note that the vacuum expectation value $\varphi_1(0)$ of the Φ_1 field on the $y = 0$ brane, with our choice of parameters gives $\varphi_1(0)\sqrt{4\pi R} \simeq 143\,\mathrm{GeV}$, very different from the corresponding $v_1 \simeq 233\,\mathrm{GeV}$ which would be obtained with the parameter choice (4.2) and $\lambda_3 = 0$. This is a 63% decrease that can modify the spectrum of the Kaluza-Klein excitations with respect to the one of the naive ansatz even if the vacuum configuration is almost constant.
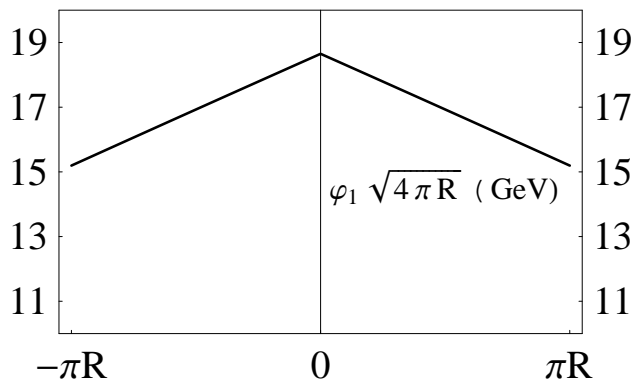
Figure 2: Vacuum configuration for the choice of parameters of section 4.2. We see that a constant φ_1 is not a good approximation: the variation from $y = 0$ to $y = \pi R$ is about 22%. The variation of $\varphi_1(0)$ with respect to the non-interacting case is even greater; we would have $\varphi_1(0)\sqrt{4\pi R} \simeq 85\,\mathrm{GeV}$ for $\lambda_3 = 0$ (with the other parameters kept constant), while $\varphi_1(0)\sqrt{4\pi R} \simeq 19\,\mathrm{GeV}$ here.

4.2 Sizable violation of translational invariance in the extra dimension

Let us allow for a discontinuity of the first derivative in $y = \pm\pi R$ assuming for $\varphi_1'(\pm\pi R)$ a given value different from 0. We make a different choice of parameters:

$$\pi R = (1\mathrm{TeV})^{-1}, \quad \mu_1 = 60 \text{ GeV}, \quad \lambda_1 = 0.5 \times 2\pi R, \quad \lambda_2 = 2, \quad \lambda_3 = 10 \times 2\pi R. \quad (4.5)$$

Again, the minimum corresponds to a B1 type solution as in eqs. (4.3) and (4.4), but with $\mu_2 \simeq 349\,\mathrm{GeV}$, $\beta \simeq 0.1$ and $y_0 \simeq 0.15\,\mathrm{GeV}^{-1}$. The energy density in this case is $\simeq -(245\,\mathrm{GeV})^4$, again indicating a spontaneous symmetry breaking. Incidentally, in this case there is also another solution, of type A, but it has a positive energy density and thus it does not correspond to a vacuum state.

Actually, and as we have commented in the introduction, boundary conditions at $\pm\pi R$ can also be interpreted as additional delta-like terms in $\pm\pi R$. Maybe the simplest possibility to impose exactly the above behavior of φ_1' in $\pm\pi R$ is to introduce in the Lagrangian a brane mass term of the form $\delta(y \pm \pi R)(-2\kappa\Phi_1^\dagger\Phi_1)$. It is easy to check that it leads to the following boundary condition: $\varphi_1'(\pm\pi R) = \kappa\varphi_1(\pm\pi R)$ that reproduces the case just described above with the choice $\kappa = 229$ GeV.

In figure 2, we show the vacuum configuration for the choice of parameters of eq. (4.5). We can note that, in this case, the constant approximation would not be appropriate, since the difference between $\varphi_1(0)$ and $\varphi_1(\pi R)$ is more than 20%. Moreover, $\varphi_1(0)\sqrt{4\pi R} \simeq 19\,\mathrm{GeV}$ while $v_1 \simeq 85\,\mathrm{GeV}$, so the corresponding relative difference is even greater than that of the previous case.

Let us conclude this section, by observing that an estimation of $\varphi_1(0)$ can be also obtained by solving the free equation of motion for the bulk scalar field with the corresponding boundary conditions and minimizing the 4D effective potential evaluated on the e.o.m. solutions. We have checked that the values of $\varphi_1(0)$ so obtained in the two examples

considered are a very good approximation of the results from the exact solutions previously discussed.

5. Summary

In this work we have shown how the explicit breaking of translational invariance on the extra dimension induced by delta-like interactions between scalar bulk and brane fields translates into the vacuum configuration. This effect modifies the naively expected pattern of spontaneous symmetry breakdown in extra dimensional extensions of the Standard Model containing such terms. In particular we have found that, if a general form for the scalar potential is considered, constant non trivial solutions of the equation of motion for the scalar fields on the bulk cannot be found. We are thus forced to consider a vacuum configuration for the scalar bulk field that depends on the extra coordinate y.

We have used a simple two-Higgs model to illustrate these effects, and, in particular, we have derived the shape of the vacuum configuration in two examples: in the first one, the y dependence is weak, so that a constant configuration may still be a good approximation; however, the value of the vacuum expectation value on the brane of the scalar bulk field is significantly shifted with respect to the case with no brane-bulk interactions, and this could cause a modification of the Kaluza-Klein spectrum of the bulk fields after the spontaneous symmetry breaking. In the second example, the y dependence is much stronger, and a constant solution would only be a poor approximation to the actual vacuum configuration.

Future developments of this work include the calculation of how the Kaluza-Klein spectrum of both the scalar and gauge fields is modified in a model with brane-bulk interactions, or how these effects modify the scattering of longitudinal gauge bosons among themselves and with Higgs bosons. In addition, one can test how a y-dependent vacuum expectation value of the Higgs field would modify the generated fermion masses.

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